Markov Chain Modelling for Sales Fluctuations of the Products

Aye Khaing Soe
University of Computer Studies(Monywa)
aye564856@gmail.com

Khin Kyawt Kyawt San
University of Computer Studies(Monywa)
kyawtkyawt7419@gmail.com

Nyunt Shwe
Technological University(Monywa)
shweshwe2510@gmail.com

Abstract

The Markov Chain is one of the most important concepts in operation research and it is used in learning the true state of nature. The stochastic processes are the probabilities models for the process that evolved over time in a probabilistic manner. In this paper, it presents the sample data which are collected from the Shwe Pyae San Company, Monywa, in details in case study. Sales fluctuations were estimated and computed by using Markov Chain. Numbers of goods in duration were conditioning calculated sales from February to April (during Coivid-19 outbreak in Myanmar). Markov Chain proves that people can predict future events ranging from the sale fluctuations to the company trend by applying the stochastic calculation correctly and analyzing empirical data from past months. The estimation using this method is useful in economy. Therefore it is observed that the products of detergent powder, soap, hand sanitizer, shampoo, etc. These are the best sellers during this period.

1. Introduction

Mathematics plays an important role in computer science and technology because it provides an easy and systematic way to model many problems. The stochastic processes are the probabilities models for the process that evolve over time in a probabilistic manner. Stochastic process is the process which comes from random value of the variable, causing statistical distribution. The analysis of this stochastic processes is Markov Chain. These problems can be expressed in terms of tables, graphs and solved by using Markov Chain Model. Markov Chain is a statistic model developed by a Russian Mathematician Andrei A Markov (1856– 1922). Markov Chain is useful tool for prediction which its extended application benefits and facilitates several areas such as, statistics, physics, chemistry, biology, genetics, meteorology, and social sciences, etc. [2] and [6].

The Markov Chain model is one of the most fundamental and intensively studied problems in marking optimization with many theoretical, practical application. It will calculate the sales fluctuations depending on the kinds of products. This paper will explore the concepts of the Markov Chain and demonstrate its applications in probability prediction area and financial trend analysis. The extrapolation of the sales of the products is important in market economy. Standard introductory texts are Hiller & Lieberman [2], Ross [3]. Furthermore, Simeyo, Nyabwanga & Otumba [1] , Shasky [4] , Svoboda [5], Myers, Wallin & Wikatorm [6], [7] and Chan & Ip [8] give a more complete recent survey of the field.

2. Case Study

The sample data from this research work collect from the sales of the products of Shwe Pyae San Company, Monywa, from February to March and from March to April, 2020. The products are consumer products such as detergent powder, soap, hand sanitizer, shampoo, toothpaste, spicy powder, cosmetics and so on. The sample comprised 50 categories such as Clear, Sun silk, Dove, Pantene, Closeup, Signal, Pond, Fair and Lovely, Rexona, Vaseline, Lifebuoy, Elan, Fuji, Lux, Sunlight, Mr. Care, Knorr, Zen, Shine, etc. According to their weight and volume, the products have various kinds of stock keeping unit (s k u) are gram, kilogram and milligram. The prices of the products are between 100 kyats and 20000 kyats if the grams of the products are 17g to 1000g. The prices of the products are between 100 kyats and 20000 kyats if the milligrams of the products are 9ml to 600ml. The prices of the products are between 100 kyats and 20000 kyats etc. In this company, the selling products are three groups. Let Group(1) be the products of cosmetics, Group(2) be the products of roll-on, toothpaste, spicy powder, Group(3) be the products of detergent powder, soap, hand sanitizer, shampoo.

3. Resources and Method

The stochastic processes are probability model for the process that evolves over time in a probabilistic manner. The special properties of Markov Chain are that the process will evolve in the future depending only on the present state of the process. A stochastic process is defined to be an index collection of random variables X_t, where index $t \in T$, T is the set of non-negative integers, [2] and [3].

A stochastic process has the Markovian property if $P\{X_{t+1} = j | X_0 = k_0, X_1 = k_1, \ldots, X_{t-1} = k_{t-1}, X_t = i\}$ $= P\{X_{t+1} = j | X_t = i\}$.

For t = 0, 1..., and every sequence $i, j, k_0, k_1, \ldots, k_{t-1}$.

A finite state Markov Chain is also often described by a matrix P.

Markovian property is denoted that the conditional probability of any future event given any past event and the present state $X_t = i$, is independent of the past event and depends only upon the present state.

A stochastic process X_t, $(t = 0,1, \ldots)$ is a Markov Chain if it satisfies the Markovian property. The conditional probabilities, $P\{X_{t+1}=j|X_t=i\}$ for a Markov Chain are called transition probabilities, for each i and j. $P\{X_{t+1}=j|X_t=i\}=P\{X_1=j|X_0=i\}$ for all t = 1, 2, ….

The properties of the Markov Chain are as follow:

(i) a finite number of states,

(ii) stationary transition probabilities,

To simplify notation with stationary transition probabilities, $P_{ij}=P\{X_{t+1}=j|X_t=i\}$.

(one step transition probabilities)

Therefore it can be stated that

$$P_{ij}^{(n)}=P\{X_{t+1}=j|X_t=i\}.$$

(n-step transition probabilities)

The n-step transition probability $P_{ij}^{(n)}$ is just the conditional probability that the system will be in state j after exactly n steps (time units), given that it starts in state i at any time t.

When $n=1$, it becomes $P_{ij}^{(1)}=P_{ij}^{1}$.

Because the $P_{ij}^{(n)}$ are conditional probabilities they must be nonnegative, $0\le P_{ij}\le 1$, and since the process must make a transition into state i to state j, they must satisfied the properties

$$P_{ij}^{(n)}\ge 0 \ \ , for\ all\ i\ and\ j\ ; n=0,1,2,\dots\ .$$

and $\sum_{j=0}^{M}P_{ij}^{(n)}=1\ , for\ all\ i=0,1,2\ \dots$.

$$\lim_{n\to\infty}P_{ij}^{(n)}=\pi_j>0\ ,$$

where π_j is uniquely satisfy the steady-state equations. Steady-state equations are

$$\pi_j=\sum_{i=0}^{M}\pi_i\,p_{ij}\ ,\quad for\ j=0,1,\dots M\ . \qquad (1)$$

$$\sum_{j=0}^{M}\pi_j=1 \qquad (2)$$

The n-step transition probabilities of the matrix form is

$$\begin{array}{cc} & \text{state}\quad 0 \quad\quad 1 \quad\quad \dots \quad\quad M \\ P^{(n)}= & \begin{array}{c} 0 \\ 1 \\ \vdots \\ M \end{array}\begin{bmatrix} p_{00}^{(n)} & p_{01}^{(n)} & \dots & p_{0M}^{(n)} \\ p_{10}^{(n)} & p_{11}^{(n)} & \dots & p_{1M}^{(n)} \\ \dots & \dots & & \dots \\ p_{M0}^{(n)} & p_{M1}^{(n)} & \dots & p_{MM}^{(n)} \end{bmatrix} \end{array}$$

The transition probabilities are from row states to column states.

The π_j are called steady-state probabilities of the Markov Chain. The term steady-state probability means that the probability of finding the process in a certain state j, after a large number of transitions tends to the value π_j, independent of the probability distribution of the initial state. The following limit always exists for an irreducible Markov Chain, [2] and [3]. $\lim_{n\to\infty}\left(\frac{1}{n}\sum_{k=1}^{n}P_{ij}^{(k)}\right)=\pi_j$, the π_j, is steady equations.

The expected average cost incurred over the n periods is $E\left[\frac{1}{n}\sum_{t=1}^{n}C(X_t)\right]$.

The (long-run) expected average cost per unit time is $\lim_{n\to\infty}E\left[\frac{1}{n}\sum_{t=1}^{n}C(X_t)\right]=\sum_{j=0}^{M}\pi_j C(j)$. (3)

For more detailed discussions of Markov Chain, especially, steady state probabilities and expected average cost can be seen in [2],[3], [7] and [8]. We define 50 types of the goods as G1, G2,…, G50.

Table1. Products Transition Data from February to April

Categories ID	Changes(from February to March)	Changes(from March to April)
G1	2	2
G2	0	0
G3	2	1
G4	2	0
G5	1	2
G6	2	0
G7	1	1
G8	2	2
G9	0	0
G10	2	2
G11	1	1
G12	2	2
G13	0	2
G14	2	2
G15	1	1
G16	2	2
G17	1	2
G18	2	0
G19	2	2
G20	0	2
G21	0	2
G22	2	0
G23	1	2
G24	0	2
G25	1	2
G26	2	0
G27	0	2
G28	1	2
G29	2	2
G30	0	2
G31	1	1
G32	2	0
G33	2	2
G34	0	2
G35	1	2
G36	2	1
G37	0	2
G38	1	2
G39	2	2
G40	0	0
G41	1	2
G42	2	2
G43	1	1
G44	2	2
G45	0	2
G46	2	0
G47	0	2
G48	1	2
G49	2	0
G50	1	2

4. Results and Discussion

Let the three states "0" , "1" and "2" be down, stable and up and these states can be changed from one state to another is tabulated as below.

Table 2. Products Transition Data

	0	1	2	Total
0	3	0	10	13
1	0	5	10	15
2	8	2	12	22

In Table 2, it can be found that there may be three groups in the products, that is Good-Sales products (GS), Sales-Stabilizing products (SS) and Low-Sales products (LS)

Group1: LS means (0, 1, 2) change to 0, i.e., the sales of the products by months are down.

Group 2: SS means (0, 1, 2) change to 1, i.e., the sales of the products by months are stable.

Group 3: GS means (0, 1, 2) change to 2, i.e., the sales of the products by months are up.

Now, LS, SS and GS products are used in Markov model for the required transition probability matrix.

So, we describe the one- step transition probability matrix according to Table 2.

$$P = \begin{matrix} & 0 & 1 & 2 \\ 0 & \frac{3}{13} & 0 & \frac{10}{13} \\ 1 & 0 & \frac{5}{15} & \frac{10}{15} \\ 2 & \frac{8}{22} & \frac{2}{22} & \frac{12}{22} \end{matrix}$$

The Markov Chain can describe the following state diagram.

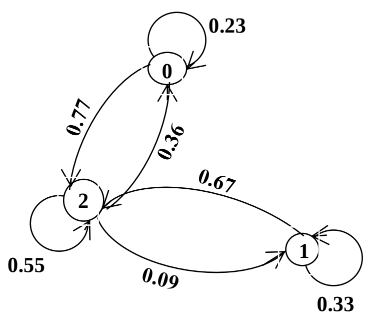

Figure1. State Diagram for Sales Fluctuations

Markov Chains are often described by a directed graph. In this graphical representation, there is one node for each state and a directed arc for each transition probability.

By the transition diagram, it can be found in the following conditions.

State (0) 0.77% of products are up and 0.23% of products are down.

State (1) 0.67% of products are up and 0.33% of products are stable.

State (2) 0.55% of products are up and 0.36% and 0.09% of products are down and stable.

To find the steady-state probability, we apply equation (1). Then a system of equations becomes:

$$\pi_0 = 0.23\pi_0 + 0.36\pi_2 \quad (4)$$

$$\pi_1 = 0.33\pi_1 + 0.09\pi_2 \quad (5)$$

$$\pi_2 = 0.77\pi_0 + 0.67\pi_1 + 0.55\pi_2 \quad (6)$$

and by an equation (2) for $j = 0, 1, 2.$

The steady-state equation $\pi_0 + \pi_1 + \pi_2 = 1$.
Therefore, this system of equations is computed and corrected up to two decimal places of π_j such that $\pi_0 = 0.47\pi_2$ *and* $\pi_1 = 0.13\pi_2$.
Then replacing these values, we get

$$\pi_0 = 0.29\,.$$
$$\pi_1 = 0.08\,.$$
$$\pi_2 = 0.63.$$

The expected average storage costs of this Company in states 0, 1 , 2 are 0, 200000mmk, 400000mmk respectively .We derive the long run expected average storage costs by using equation (3)

$$\lim_{n\to\infty} E\left[\frac{1}{n}\sum_{t=1}^{n} C(X_t)\right] = \sum_{j=0}^{M} \pi_j C(j)$$

$$= \pi_0 c(0) + \pi_1 c(1) + \pi_2 c(2)$$

$$= 268000 mmk.$$

When Markov Chain Model calculates the sales of the products, the sales of the products are up to 63%, the sales of the products are down to 29%, the sales of the products are stable at 8% respectively. The products which are on good sales are detergent powder, soap, hand sanitizer, shampoo, etc., and the products which are on stable sales are toothpaste, spicy powder, etc. the products which are on low sales are cosmetics, roll-on, etc. In this company (the expected average cost), actual average cost per unit month is 268000mmk.

After we have analyzed the sales fluctuations of the product of that company, it is found that from February to March, the sales of some products are down to 26% , the sales of some products are stable at 30%, the sales of some products are up to 44% and from March to April, the sales of some products are down to 22%, , the sales of some products are stable at 14% , the sales of some products are up to 64%. When the fluctuations of these products are analyzed, it is found that the sales of the detergent powder, soap, hand sanitizer, shampoo are up, the sales of the cosmetics are down, the sales of the roll on, toothpaste, spicy powder are stable.

Since the probability of the selling products, future states only depend on the present states. Therefore, Markovian property holds for the evaluation of sales fluctuations.

Here, we illustrate the results as the following column chart diagram for the values of sales fluctuations.

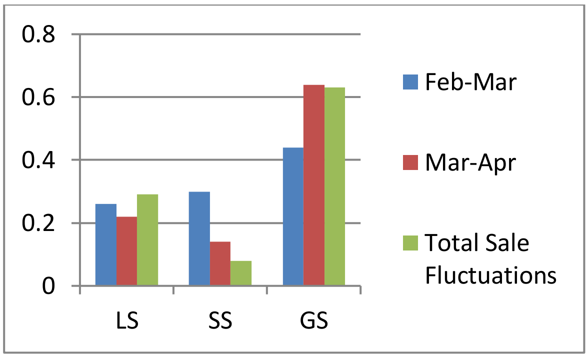

Figure.2 Comparison Values of Sales Fluctuations

5. Conclusions

The main purpose of this research paper is to use this model in applied research in other fields, using theoretical ideas of Markov Chain. Researches may get some information related to this model and its applications in economic fields and can get some ideas related to their fields of research. During Covid-19 outbreak (especially February-April), it is found that Group(3) is the best seller among all of groups after having analyzed the data of the company by using mathematical sciences. So, to conclude this research paper, it is discovered that Group (3) may be the best sellers throughout the whole Myanmar and other countries. In the same way, the sales of other products can be calculated by using Markov Chain Model. It is guessed that this process may be the same as the one from other places.

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